Social Interaction and the Dynamics of Crime

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1. Introduction:

In recent decades, crime rates have increased dramatically in many Western countries and crime has become a major issue of public policy, especially in the United States. A very large literature has developed on the general causes of, and the impact of public policy on, crime. Yet no consensus has emerged on quite basic issues such as, for example, the effect of incentives, both positive and negative, on crime. Leading conservative criminologists such as Wilson (1975) argue the case for the effectiveness of the policies of deterrence, and even though his original position has softened somewhat (Wilson 1994), the conclusions which he draws from the evidence are still quite different from those of liberals such as Currie (1985, for example).

Economic analysis of the phenomenon of crime was stimulated in the late 1960s by the paper by Becker (1968). In this framework, agents in the market for crime - for example, criminals and the law enforcement agencies - are assumed to act in accordance with the rules of optimising behaviour. They are able to both gather and process substantial amounts of information efficiently in order to form expectations on the likely costs and benefits associated with different courses of action, and to respond to incentives and disincentives in an appropriately rational manner. This is in marked contrast to the tenets of conventional criminology (for example, Nuttall 1994 and Gottfredson and Hirshi 1995), in which the typical criminal is portrayed as having difficulty identifying and assessing alternative courses of action, rarely thinking through the consequences of actions, and not thinking about possible punishments.

But even amongst research carried out within the common framework of economic theory few clear conclusions emerge. Ehrlich (1996), in an excellent exposition of the approach to crime based upon such theory notes that the empirical literature is 'voluminous'. However, he states that on the crucial question of the impact of positive and negative incentives on crime, 'it would be premature to view the empirical evidence as conclusive'. Ehrlich notes that the quantitative estimates of such effects vary, even to the extent of a minority of studies failing to find any effect at all, and part of his paper is devoted to a discussion of the potential reasons for this, not least of which is the intrinsic limitations of crime statistics. Indeed, an awareness of the often seriously unreliable nature of the statistics is one thing on which most economists and criminologists find common ground.

In this paper, we offer a different methodological approach to the process by which crime rates spread (or contract) over time. Our aim is not to account for the path over time followed by any particular set of crime rates, whatever the degree of reliability which might be attached to any available data. In principle, the model could be calibrated to a particular data series, but our purpose is to give a general description of how crime rates change over time.

The approach is similar to that used in mathematical biology to describe how potential epidemics are either spread or contained in a population (see, for example Murray 1990). We divide the population at any point in time into a small number of discrete groups which differ in their potential to commit crime. There is a set of flows between these groups, whose overall effect describes the evolution of crime rates. The flows are postulated to depend upon key elements identified in the literature, such as the impact of incentives or general social and economic conditions.

An essential element in this model is social interaction between individual agents. The model is developed at the aggregate level, describing the outcome of decisions made by individual agents of whatever degree of (bounded) rationality might be thought appropriate. Individuals form views on what we might call external factors, such as the overall social and economic conditions and the punishment structure, and use these to determine their movement or otherwise in or out of the different categories of agent in the model. But they are also influenced by the behaviour of others, in two senses.

First, the greater the proportion of agents in any given population who are already criminals, the more likely it is that any other individual will convert into becoming a criminal. Second, the greater the proportion of the population who are wholly disinterested in being criminals, the greater the pressure on those who are criminals to become law-abiding.

The importance of social interaction for overall outcomes is one which is often stressed in the conventional criminology literature and, more generally, within the discipline of sociology. Recently, economists have begun to develop more formal models which take this into account, and the paper by Glaeser et.al. (1996) provides empirical evidence of the importance of social interaction. In a number of ways our model is similar to that of Glaeser et.al., but there are important differences which are documented in section 3 below.

Section 2 gives an overview of the model, and section 3 sets out the formal mathematical specification, along with a description of the criteria we use in order to evaluate the model. Section 4 discusses its properties, and in particular the sensitivity of crime rates to the various factors in the model.

2. The dynamics of crime: an overview

We can think of the population as being divided conceptually into three groups. First, those who are not susceptible to commit a crime (denoting this group subsequently as N, for 'not susceptible'). In other words, individuals with a zero probability of committing a crime. As a not unreasonable approximation, for example, most women, certainly those over 25, might be placed in this category, as might most pensioners. We should stress, however, that the word ‘population’ does not necessarily refer to the population of an entire nation, and could equally well apply to that of an individual city or even a neighbourhood within a city.

The second group is made up of the susceptibles (S), those who have not yet committed a crime, but might well do so. It is very well documented in the criminology literature that young men in their teens and early twenties are particularly prone to commit crimes (see, for example, Austin and Cohen (1995) and DeIulio (1996)). Of course, by no means all men in this age group actually commit crimes, but there is a high propensity to do so, from acts of minor vandalism carried out in what used to be known as 'high spirits', to brawling in public, through to far more serious crimes.

The third group is made up of those who are active criminals (C). These three groups, N, S and C by definition make up the whole population.

Of course, modelling the proportion of any given population in the Criminal category does not necessarily describe the evolution of crime rates over time. In some areas, such as vehicle crime, the increase in opportunity afforded by the spread of car ownership has led to the typical car criminal committing many more offences per unit of time. But a description of how the number of crimes committed by each criminal per unit of time evolves over time could easily be added to the model to give this information. Our concern is to describe the processes which determine the proportion of the population who are in the criminal category at any point in time.

There are undoubtedly many factors which govern both membership of and movement between the various categories over time. Some will be of lasting influence, and some will be ephemeral. The aim of the model is to synthesise the key elements which give rise to changes in the relative size of these groups over time, in order to improve our understanding of the dynamics of the behaviour of crime.

An essential part of the model is the influence of social interaction on the behaviour of agents. The importance of this phenomenon is well documented in the literature of criminology and, more generally, in that of sociology itself. Currie (*op.cit.*), for example, discusses the very wide variations in crime rates which are often observed between the rural and urban sectors of poor economies, attributing in large part the low rates in the rural areas to the community relationships which both foster a sense of belonging and provide 'the setting in which informal social sanctions against aggression and crime can operate effectively'.

We also introduce potential influences from demographics, overall social and economic conditions, and the deterrence effects of the criminal justice system. These appear to be the key external factors identified by the literature, even though, to stress again, the literature is very divided as to the relative importance of these variables.

We represent these elements by means of a small number of distinct flows between the three groups of the population. The flow from Non-susceptible to Susceptible is postulated to be due primarily to demographic movements in the population.

There is a reverse flow from Susceptible back into Non-susceptible, and three main factors are assumed to underlie this flow. First, demographics may again be important. Some young men, for example, pass through the years when they are particularly susceptible to commit crimes without actually doing so. Second, we should allow here for the possibility that deterrence - or 'negative incentives' in the economic literature - may act as a factor in removing people from the susceptible category. Third, favourable social and economic conditions may induce people to move back into the non-susceptible category.

There is also a flow out of the Susceptible category and into the Criminal. We assume that part is due to the net effect of social and economic conditions and deterrence. The flow also arises because of social interaction, with susceptible individuals being influenced by the behaviour of those who are already criminals.

Finally, there is a flow from the category of Criminals to the Non-susceptibles. Again, in part a number of people abandon crime in response to the net effect of social and economic conditions and deterrence. In part, the extent of general social disapproval of criminal activity is also postulated to determine this flow.

As an overview, we have the following flows in the model between the three categories of the population, along with our interpretation of the main factors underlying each of them:

From Non-susceptible to Susceptible:
demographics

From Susceptible back into Non-susceptible:
demographics, deterrence, social and economic conditions

From Susceptible into Criminal:
i) deterrence, social and economic conditions
ii) social interaction

From Criminal to Non-susceptible:
i) deterrence, social and economic conditions
ii) social disapproval

The model could be extended by, for example, splitting the criminal category into two, namely the occasional and the habitual criminal. Following the seminal work of Wolfgang (1972), it has become well established that a small minority of criminals are responsible for a very disproportionately larger percentage of crime committed. Wolfgang, in a study of youths in Philadelphia, found that 6 per cent of boys were responsible for over half the total crimes committed by their age cohort. Nuttall (*op.cit.*), surveying the subsequent literature, confirms Wolfgang's initial finding and states that 'Six per cent of offenders are responsible for between 65 and 75 per cent of known offences'. A further, related, extension would be to include explicitly the proportion of the population held in prison at any one time

But these extensions would not really add a great deal to the insights which this system provides into the dynamics of crime, for two reasons. First, the proportion of the population in the 'habitual' and 'prison' categories at any point in time is small relative to those in the N, S and 'occasional criminal' categories, and so many of the main properties of the extended system can be understood by first analysing the latter three in isolation.

Second, to a large extent the habitual and prison categories would form a self-contained loop in the system. It would not be unreasonable to assume as an approximation that no-one moves directly from the N or S categories directly into the habitual category, and that the flow into this latter comes exclusively from the 'occasional' category. Similarly, as an approximation the flow into prison only comes from habitual criminals, since it is they who, by either persistence or seriousness of offence, are far more likely to merit it than are occasional criminals. Further, given the high rates of recidivism, a substantial part of the flow out of prison is back directly into the habitual category.

As noted above, a key feature of the model is social interaction. A small number of more formal models of the process of crime have already been developed which take this into account in analysing crime, such as Sah (1991), Murphy *et.al.* (1993) and Glaeser *et.al.* (*op.cit.*). The first two papers rely upon very specific assumptions in order to be able to generate multiple solutions to the mathematical equations. In Sah, the choice of any individual to become a criminal lowers the probability that any other criminal will be arrested. On the assumption that the police cannot arrest more than a fixed number of criminals, when crime increases the probability of arrest falls. In Murphy *et.al.*, high levels of crime crowd out legal activities so, for example, when the number of criminals rises, the returns from not being a criminal fall because legal revenues are stolen by criminals.

Glaeser *et.al.*, whilst welcoming the use of social interaction terms, criticise these two papers on the grounds that multiple equilibria models should predict that the data for, say, crime rate across cities in the US, should be clustered into a small number of distributions corresponding to the number of equilibria. They find that the actual data shows considerably more variance than this implies. The criticism only appears valid, however, if it is assumed that the parameters of any particular model are fixed across cities. *Prima facie*, there is no reason to believe that the parameters of a mathematical model describing, for example, crime in Chicago should be the same as those describing crime in New York.

The Glaeser *et.al.* model is based upon the behaviour of individual agents at a very local level, in which there are three types of agent. The first two are those who are die-hard lawbreakers and those who are diehard law-abiders, neither of which are influenced by the actions of others. The third category imitates the behaviour of his immediate neighbour. The authors state that the first two types are maximising utility by making their choice, but this is not strictly relevant to their model. In one sense, the spirit of their model is certainly in keeping with our own in that it is not necessary to specify any of the factors which determine whether or not, for example, an individual chooses to be a diehard criminal. The model analyses the consequences once such a decision is made.

In our model the social interaction is postulated to work at the aggregate level, with the total proportion observed in the C category influencing the decision of people to convert from S, but as Glaeser *et.al.* note 'ideally, a model might contain both local interactions and global interactions'.

An essential difference between the two, however, is in that our model, implicitly, any individual agent can be influenced by social interaction. The Glaeser *et.al.* model rests on the assumption that there is a proportion of the population who are potential criminals who do not respond under any circumstances to such interaction. Indeed, one of the main purposes of their paper is to try to identify this proportion across the cities and precincts of the United States. In our model, although in equilibrium the proportion of the population in each category is fixed, flows between the categories continue to happen. Implicitly, even hardened criminals are redeemable and can be turned from crime.

3. The formal model:

In this section we develop a formal mathematical model of the flows described above. For the most part, in the absence of good evidence to the contrary, we make the simplest possible assumption, namely that the flows into and out of any particular group are proportional to its size. This is perfectly justifiable, since the lack of conclusions in the empirical literature on the impact on crime of the factors listed above means that there is little guidance as to how we should express these factors in a formal, mathematical way.

Consider first of all the flow from the non-susceptible into the susceptible category. We mentioned above the demographic factor of the proportion of young men in a population as a potential key determinant of crime rates, and we interpret the strength of this particular flow in our model to be determined by demographics. The assumption that a constant proportion of the Non-susceptibles flows into the Susceptible category leads to the differential equation for N,

$$dN/dt = -\theta N$$

where θ is a constant which measures the size of the flow. The constant θ must be positive if it is to represent a flow out of N.

There are several other flows into and out of the Non-susceptible category that must be included in the equation for N. There is a flow from the Susceptibles back into the Non-susceptibles which we take to be a constant proportion of S. This gives rise to a term μS in the equation for dN/dt where the constant μ measures the size of the flow, and it too is assumed to be positive.

As discussed above in section 2, there are three main factors assumed to underlie this flow from S to N, and the single parameter, μ, captures their overall net effect. The relative weight which one assigns to these three factors will obviously depend upon one's interpretation of the diverse empirical results which exist.

There is also a flow from the category of Criminals to the Non-susceptibles. The pressure on criminals to drop out of the Criminal category will depend not just (proportionally) upon the net effects of deterrence and social and economic conditions, but also on the extent to which it is the object of social disapproval. For the sake of simplicity, we express social disapproval as a function of the numbers who are in the Non-susceptible category at any point in time. But the impact of disapproval is unlikely to be one of simple proportionality to N. Where N is already high, further increases may well have little effect in influencing the small number who are criminals, whilst for low values of N, in areas where crime has almost become the norm, further declines may also have little effect.

These effects are captured by the function β which is defined by

$$\beta(N) = \beta_1 + \beta_2 / (1 + \exp(-\rho(N - N_c))).$$

The parameters β_1, β_2 etc. need some explanation. For low values of N, the value of β is approximately β_1 which therefore represents the net effects of deterrence and social and economic conditions. The effects of social disapproval are only felt for sufficiently high values of N when the value of the function is approximately $\beta_1 + \beta_2$. The other parameters N_c and ρ determine respectively the value of N for which social disapproval becomes significant and the sensitivity of disapproval to changes in the value of N. A plot of the function is shown in figure 1 for the parameter values $\beta_1 = 0.1$, $\beta_2 = 0.3$, $\rho = 25$, $N_c = 0.75$. We assume that $\beta(N)$ is the proportion of the Criminals flowing into the Non-susceptibles.

The equation for N can then be written in full,

$$dN/dt = -\theta N + \mu S + \beta(N)C \qquad (1)$$

where C denotes the proportion in the Criminal category.

The equation for the Susceptibles, S, is derived in a similar way. Part of the flow out of the susceptible category into crime itself is due to social and economic conditions, but that this is potentially reduced by the impact of deterrence. In addition, we allow for the effect of social interaction. The greater the proportion of a population who are already criminals, the more likely it is that the process of social interaction will encourage others to move from being merely susceptible to crime to committing crime.

The equation for S is therefore given by,

$$dS/dt = \theta N - \mu S - \alpha S - \lambda SC \qquad (2)$$

All of the terms in the remaining equation, the equation for C, have already appeared in equations (1) and (2) but they appear here with their signs changed, i.e.

$$dC/dt = \alpha S + \lambda SC - \beta(N)C. \qquad (3)$$

Our model is completed by the identity which states that the population, however it is defined, is comprised of the sum of the proportions in the three categories:

$$N + S + C = 1 \qquad (4)$$

The proportions could be applied to a population to give the numbers in each of the categories if necessary and the population could be either constant, growing or even falling. Equations (1) to (4) taken together formalise the model.

Each of the parameters in the equations - α, β and so on - is meant to reflect the impact of one or more key factors which influence the process of crime. The relative importance of the factors which underlie them are a matter of one's interpretation of the empirical literature on crime. Consider, for example, the αS term in the dS/dt equation. The parameter α is intended to capture the net effect of the impacts of deterrence and general social and economic conditions (negative and positive incentives) on the decision to become a criminal. The literature gives scope for a wide variety of views on the relative importance of these two effects. Therefore when it comes to choosing a value for this parameter, its value could be interpreted as representing the effect of deterrence if we felt that this largely outweighed social and economic conditions. On the other hand, if we made the judgement that deterrence had little effect, α would be interpreted as the effect of social and economic conditions. Once this is decided, the effect of varying the size of α relative to the other parameters shows the effect of varying the relative importance of this particular link in the system. For example, increasing its value compared to, say, μ, examines the consequences of the general factors which lead to an outflow from Susceptibles into Criminals becoming stronger compared to those factors which lead Susceptibles to move back into the Non-susceptible category.

There are several points to bear in mind when discussing the success or plausibility of the model. We are not, to emphasise again, attempting to account for any particular set of observed crime rates, so the standard statistical criteria of econometric work, for example, are not relevant. (It should also be said, however, that given the failure of such work to produce anything like a consensus on the determinants of crime, the apparent rigour of such tests in theory is very much weaker in practice). We do not attempt to disguise the fact that there are no formal criteria and that ultimately the plausibility of the model is a matter of judgement, but we set out a number of criteria which it should satisfy.

The underlying rationale of the model implies, for example, that a higher value of α, say, or of θ should eventually lead to a higher level of crime - to a greater proportion of the population being in the Criminal category. So it is essential that the model as a whole preserves these qualitative properties which underlie each individual expression in it. It is by no means guaranteed that the system of coupled nonlinear differential equations will behave in the way we might expect from each single equation considered in isolation.

A second, very important, criterion of plausibility is that the system should be able to generate a wide range of solutions particularly for the value of C, with respect to relatively small variations in the values of the parameters. An important stylised fact about crime rates is their enormous variation, both over time and, especially, over space. This latter holds not just in terms of international comparisons, but within individual countries. The variation appears to be considerably more substantial than that of factors which might be thought of as possible contributors to the crime rate. So reasonable variations in the values of the parameters should be able to produce a wide range of solutions.

Third, the model must give rise to solutions which are meaningful, in the sense that since N, S and C are proportions of any given population, their values cannot be less than zero, nor greater than one. Therefore, starting from any meaningful set of initial values, the model must not be able to generate solutions which, whilst valid mathematically, nevertheless violate this restriction.

It is more difficult to judge the significance of the value of any individual parameter as such, not least because of the alarming lack of consensus in the standard literature on the effects of various factors on crime rates. However, it would be a weakness of the model if reasonable solutions could only be generated if one or two of the links were very much stronger than the rest. If this were the case in actual reality, we would expect the existing literature to have come to something more of a consensus than is actually the case.

4. Analysis of the mathematical model

The formal mathematical model is given by the system of equations (1) - (4) and as these are differential equations, their solution is a set of functions of time N = N(t), S = S(t) and C = C(t) for all values of time t greater than some suitable starting point $t = t_0$.

In studying a system of differential equations we want to understand how the solutions behave over time and how they depend on the values of the parameters α, β, λ etc. As we will see, these equations have an attracting limit point, i.e. there is a value C_* such that C(t) tends to C_* as t increases regardless of the initial value for C. The other variables S and N also tend to a limiting value.

The first step in analysing this system of equations is to verify that the solutions are consistent with our interpretation. Since N, S and C represent proportions of the population in different groups, they should always sum to 1. That is to say,

$$N(t) + S(t) + C(t) = 1 \qquad (5)$$

should hold for all values of time t. This can be verified easily. We assume that the initial values of the variables sum to 1, i.e.

$$N(t_0) + S(t_0) + C(t_0) = 1.$$

If we take N(t) + S(t) + C(t) as a single quantity and differentiate it with respect to time t, its derivative is 0 for all values of t. This is no more than to say that the terms on the right hand sides of equations (1) - (3) cancel out. Therefore the value of N(t) + S(t) + C(t) is constant over time.

It is another matter to show that each of the variables lies between 0 and 1. As a first step in this direction, we simplify the system of equations (1) - (4) by eliminating the variable N. On substituting $N = 1 - S - C$ into equations (2) and (3) we obtain

$$dS/dt = \theta(1 - S - C) - \mu S - \alpha S - \lambda SC \qquad (6)$$

$$dC/dt = \alpha S + \lambda SC - \beta(1 - S - C)C \qquad (7)$$

A solution to this system of equations then gives a solution of equations (1) - (3) by putting $N(t) = 1 - S(t) - C(t)$.

These equations can be made more simple again by eliminating one of the parameters. We do this by setting $\theta = 1$ which gives the equations

$$dS/dt = 1 - S - C - \mu S - \alpha S - \lambda SC \qquad (8)$$

$$dC/dt = \alpha S + \lambda SC - \beta(1 - S - C)C \qquad (9)$$

The solution to the system of equations with $\theta \neq 1$ can be derived from the solution for $\theta = 1$ with modified values of the other parameters as follows. Suppose that $S' = S'(t')$, $C' = C'(t')$ is a solution of equations (8) and (9) for the parameters $\alpha' = \alpha/\theta$, $\lambda' = \lambda/\theta$, $\mu' = \mu/\theta$ and $\beta' = \beta/\theta$. By simply carrying out the calculations for their derivatives, it can be seen that $S(t) = S'(\theta t) = S'(t')$ and $C(t) = C'(\theta t) = C'(t')$ give a solution of the original equations (6) and (7) for the original parameter values α, λ etc.

Now we return to showing that the solutions of the equations lie between 0 and 1. This requires that the solutions S(t) and C(t) to equations (8) and (9) both lie between 0 and 1 and in addition that S(t) + C(t) is less than 1 so that $N(t) = 1 - S(t) - C(t)$ also lies between 0 and 1. This can be summarised neatly in a geometrical way by saying that the point (S(t), C(t)) must lie in the triangular region T of the plane defined by

$$T = \{(S, C) : 0 \leq S \leq 1, 0 \leq C \leq 1 \text{ and } 0 \leq S + C \leq 1\}$$

for all values of t.

It will help in understanding systems of differential equations and their solutions to introduce the idea of a *vector field*. A vector field is a function or rule that associates a vector with each point in the plane in exactly the same way that a function of one variable associates a value or number with each point on the line.

A vector field can be plotted. Suppose that a vector field denoted by ϕ is defined on some region of the plane, and choose a grid of points that covers the region. At each point x on the grid, the value of the vector field $\phi(x)$ is a vector. To plot the vector field, draw an arrow representing the vector $\phi(x)$ from the point x. (Depending on how you think of a vector, the arrow either represents or *is* the vector $\phi(x)$. The arrow points in the direction of the vector and its length is equal to the norm of $\phi(x)$.) The pattern of the vectors plotted in this way gives a good indication of the vector field.

The system of differential equations (8) and (9) is equivalent to a vector field. The vector field associates the point with coordinates (S, C) in the plane with the vector (dS/dt, dC/dt). Any system of differential equations can be expressed in this way. A solution of the equations (S(t), C(t)) traces out an unbroken curve in the plane as t increases. It has the property that at each point (S, C) on the curve, the tangent to the curve is the value of the vector field at that point, i.e. (dS/dt, dC/dt).

Viewing the equations as a vector field it becomes clear that in order to show that a solution with valid initial values remains in the region T, we simply need to verify that at each point on the boundary of T the vector field given by (dS/dt, dC/dt) points towards the interior of T. This ensures that a solution that starts in the region T stays inside T since it can not cross its boundary.

We can now examine each section of the boundary of T in turn. The section that lies on the S-axis has coordinates C = 0 and $0 \le S \le 1$. From equation (9) it follows that

$$dC/dt = \alpha S.$$

Provided $0 < S < 1$, $dC/dt > 0$ since we assume $\alpha > 0$ and therefore the vector (dS/dt, dC/dt) points upwards towards the interior of T regardless of the value of dS/dt. The values S = 0 and S = 1 must be looked at more closely. When S = 0 (and C = 0), the vector (dS/dt, dC/dt) is equal to (1, 0) which runs along the S-axis so that the solution still remains inside T. At the other point where S = 1, (dS/dt, dC/dt) is equal to $(-\mu - \alpha, \alpha)$ and since $\mu > 0$, this vector points below the line given by S + C = 1 which forms part of the boundary of T.

Similar calculations show that the vector field (dS/dt, dC/dt) lies either along the boundary or points towards the interior of T on the two remaining sections of the boundary of T. Therefore given valid initial values (S_0, C_0) the solution to equations (8) and (9) lies in the region T and can be interpreted as representing proportions of a population.

Now we turn to the behaviour of the equations as a function of time. We first look for equilibrium points, that is points (S, C) where the vector field is zero or equivalently dS/dt = 0 = dC/dt. It is difficult to solve such equations analytically so we rely on numerical approximations.

Before tackling equations (8) and (9) as they stand, we try a number of simpler cases. Although the simplified equations are no longer suitable formalisations of the model set out in section 2, their properties do indicate the likely behaviour of the full equations. Analysing the simpler cases also provides hints as to how to solve the more general equations.

When β is a constant and the parameter α is zero, the equations can be solved quite easily. It can be shown that in this case there is an attracting equilibrium point, i.e. a point (S_*, C_*) such that $(S(t), C(t))$ tends to (S_*, C_*) as t increases regardless of the initial value $(S(0), C(0))$ of the solution. The steps involved in demonstrating this are not very difficult and are similar to those given below for a slightly more general case.

We now allow α to be non-zero but still assume β is constant. To find an equilibrium point we must solve the equations

$$dS/dt = 1 - S - C - \mu S - \alpha S - \lambda SC = 0$$

$$dC/dt = \alpha S + \lambda SC - \beta C = 0$$

for both S and C. This can be done by solving each separate equation for C in terms of S to give

$$C = (1 - (1 + \alpha + \mu)S) / (1 + \lambda S) \qquad (10)$$

from the equation $dS/dt = 0$ and

$$C = \alpha S / (\beta - \lambda S) \qquad (11)$$

on putting $dC/dt = 0$. Equating these two expressions for C leads to a quadratic equation for S,

$$\lambda(1 + \mu)S^2 - (\lambda + \beta(1 + \mu + \alpha) + \alpha)S + \beta = 0. \qquad (12)$$

The nature of the roots of the quadratic equation $aS^2 + bS + c = 0$ is determined by the discriminant $\Delta = b^2 - 4ac$. If $\Delta > 0$ then there are two real valued solutions, if $\Delta = 0$, just one real valued solution but if $\Delta < 0$ then the solutions are complex rather than real valued. The discriminant of equation (12) is given by

$$\Delta = (\lambda + \beta(1 + \mu + \alpha) + \alpha)^2 - 4\lambda\beta(1 + \mu). \qquad (13)$$

The first of these terms can be rearranged to give

$$(\lambda + \beta(1 + \mu))^2 + \text{other terms}$$

where the other terms are sums and products of the parameters α, β, λ and μ which are all positive. The term $(\lambda + \beta(1 + \mu))^2$ and the second term on the right hand side of equation (13) can be combined and simplified to give $(\lambda - \beta(1 + \mu))^2$ (the + sign has changed to a − sign). The discriminant can now be written

$$\Delta = (\lambda - \beta(1 + \mu))^2 + \text{other terms.}$$

Since the squared term and the other terms are both positive, so too is the discriminant. Therefore there are two real valued solutions for S in equation (12).

For each solution S of equation (12) there is a corresponding value of C that is given by substituting the value for S into either equation (10) or (11). This yields two points (S_1, C_1) and (S_2, C_2) that are equilibrium points of the vector field, i.e. points where the value of the field is the zero vector. The solution to the differential equations with the initial value (S_1, C_1) is therefore constant, i.e. $S(t) = S_1$ and $C(t) = C_1$ for all values of t.

Only one of the equilibrium points falls in the region T and is valid for our interpretation of the model. The C co-ordinate of the other equilibrium point is negative so that it lies well outside this region. (Although this does not follow immediately from the analysis shown above, it will be borne out by the examples given later.) Given that any solution with its initial value in T remains inside the region, we might expect the equilibrium point that does lie in T to be an attracting point.

As we will see, when β is not held constant, equations (8) and (9) can have both stable equilibrium points and saddle points.

The stability properties of an equilibrium point can be determined by making a linear approximation to the actual equations and then examining the stability of the linear approximation. The stability properties of linear differential equations can be worked out very easily.

Suppose we have the linear differential equation

$$dx/dt = Ax \qquad (14)$$

where x is now a vector in two dimensions, A is a 2×2 matrix and on the right hand side Ax is the product of the matrix A and the vector x. It is clear that the origin (0, 0) is an equilibrium point for equation (14) and provided that A has full rank, it is the only equilibrium point. The stability properties of the origin depend in a very simple way on the eigenvalues of the matrix A. If both the eigenvalues of A have negative real parts, then solutions that start near the origin are drawn closer to it and the origin is an attracting equilibrium. On the other hand, if both eigenvalues have positive real parts, it is an unstable equilibrium point. The only remaining case is when the real part of one eigenvalue is positive and the other negative. This makes the origin a saddle point and the stable axis is in the direction of the eigenvector corresponding to the eigenvalue with negative real part, the other eigenvector determines the unstable axis since its corresponding eigenvalue has positive real part.

We now apply this technique to analysing the stability properties of the equilibrium point that lies in T. Denote this point by (S_*, C_*). We need first to make a linear approximation to the vector field that holds good in the neighbourhood of (S_*, C_*). The vector field can be regarded as a map from two dimensional space to two dimensional space. The derivative of this map evaluated at the point (S_*, C_*) gives the linear approximation we are looking for. The derivative can be written as a matrix D_*. (This an extension to two dimensions of the idea that the derivative of a function f of one variable at a point x is the slope of the line tangent to the graph of the function at the point x. The line tangent to the graph is a linear map or function approximating the function f in the neighbourhood of x.) Taking the derivative of the vector field is not to be confused with the fact that these equations already involve a derivative, namely a derivative with respect to time. The derivative of the vector field involves the partial derivatives of each of the right hand sides of equations (8) and (9) with respect to the variables S and C.

When the derivative of the vector field is calculated, the eigenvalues of the matrix D_* immediately determine the stability properties of the equilibrium point (S_*, C_*). The eigenvalues of the matrix both have negative real parts so that the equilibrium point is stable. (This was also confirmed by numerical solutions of the differential equations, regardless of the initial value, the solution tended to (S_*, C_*) with increasing time t.)

Now we consider the general case where β is a function of the proportion of non-susceptibles $N = 1 - S - C$. When β is no longer held constant the behaviour of the equations is much richer. It is possible that as one of the parameters in equations (8) and (9) is increased while the others are held fixed, two other equilibrium points appear, one of which is stable while the other is a saddle point. As the parameter is increased further two of the equilibrium points coalesce leaving a single stable point again, in other words a bifurcation exists.

Finding the equilibrium points for the full set of equations is quite a bit more difficult than the case we have just considered since it involves solving the equations

$$dS/dt = 1 - S - C - \mu S - \alpha S - \lambda SC = 0$$

$$dC/dt = \alpha S + \lambda SC - \beta(1 - S - C)C = 0$$

for S and C where β is now a function rather than a constant. The point (S_*, C_*) will be an equilibrium point for equations (8) and (9) if it happens to coincide with the equilibrium point for the case where β is constant and takes the value $\beta = \beta(1 - S_* - C_*)$. We can use a graphical method to find the point where they coincide.

Suppose for the moment that N is fixed. The value of N determines the value taken by the function β. For any constant β we can find an equilibrium point (S_*, C_*). Let $(S_*(N), C_*(N))$ be the equilibrium point for the value of β given by N. The value of N also determines a line in T, i.e. $N = 1 - S - C$ can be rearranged to give C in terms of S,

$$C = (1 - N) - S$$

which is the equation of a line with a slope of -1 intercepting the C-axis at $1 - N$. We have found an equilibrium point if $(S_*(N), C_*(N))$ lies on this line.

We define a function γ to measure the vertical distance between the point and the line,

$$\gamma(N) = C_*(N) - ((1 - N) - S_*(N)).$$

By plotting $\gamma(N)$ against N, the values N_* for which $\gamma(N_*) = 0$ can be found. The equilibrium points are then the points $(S_*(N), C_*(N))$ corresponding to $N = N_*$.

Plotting γ also allows us to see the bifurcation described above. Figure 2 shows a plot of $\gamma(N)$ against N for two different values of the parameter α while the other parameters are held fixed. The solid line shows the value of $\gamma(N)$ when $\alpha = 0.05$ for values of N from 0.5 to 0.9. There is a single equilibrium point corresponding to $N \approx 0.81$ where the solid line crosses the horizontal axis. The dotted line shows $\gamma(N)$ for $\alpha = 0.075$, this is very similar to the curve for $\alpha = 0.05$ except that it is now slightly higher. There are now three equilibrium points corresponding to $N \approx 0.58$, 0.67 and 0.79. The stability properties of these points can be determined just as for the case considered above however we must now make a linear approximation to the full equations (8) and (9) and allow for the fact that β is no longer constant but a function of S and C. This introduces a few extra terms when the partial derivatives are calculated. The equilibrium points corresponding to the values $N \approx 0.58$ and 0.79 prove to be attracting and the third equilibrium point proves to be a saddle point.

5. Evaluating the model

The first point to note is that the effect in the overall model of varying the parameters is consistent with what we would expect from the underlying logic of the individual equations. As θ, the strength of the flow from N to S, is increased, the equilibrium level of crime C_* rises. The same is true of the parameters α and λ. If μ, the rate at which Susceptibles return to the category of Non-susceptibles, is increased, C_* falls. Increasing the effects of social disapproval by increasing the values taken by the function β also leads to a fall in the equilibrium level of crime.

Second, and importantly, the model is consistent with the key stylised fact about crime rates. Namely that the variation across time and space appears to be much more substantial than that of external factors such as social and economic conditions which might be thought of as possible contributors to the crime rate. A very wide range of values for $C*$ can be generated by the model when just one or two of the parameters are varied, sometimes by relatively small amounts.

The existence of such a wide range of solutions arises from the bifurcation properties of the model discussed in section 4 above, which in turn are due to the nonlinearities introduced by the social interaction terms in the model. In other words, it is the social interaction terms which give the model its most distinguishing characteristic.

The properties of the model can be illustrated by numerical simulations. We first establish a suitable set of values of the parameters as a base case. We make a simple approximation to guide our choice of the parameter μ. If we ignore C, then in equilibrium the flows between N and S must cancel. Equating these flows gives $N = \mu S$ and therefore if S is to be about 20 per cent of the population, we can take $\mu = 5$. The proportion in the criminal category depends on the balance between the parameters α and λ on the one hand and the function β on the other. If β is too large, C is reduced to zero which is clearly unsatisfactory. The choice of the function plotted in figure 1 and the values $\alpha = 0.05$ and $\lambda = 0.7$ was made after some experimentation.

These parameters give a single stable equilibrium point with $N_* = 0.80$, $S_* = 0.16$ and $C_* = 0.04$, that is to say the criminal category accounts for 4 per cent of the population. These proportions are certainly realistic for many of the situations the model is intended to describe.

Figure 3 shows the bifurcation that occurs as α is varied while the other parameters are held fixed. The equilibrium values of C_* are plotted, the solid lines indicate the stable equilibrium points and the saddle point is shown as a broken line. The value of α ranges from 0.02 to 0.14 while the others have the values given above, i.e. $\beta_1 = 0.1$, $\beta_2 = 0.3$, $N_c = 0.75$, $\rho = 25$, $\lambda = 0.7$ and $\theta = 1$. This illustrates a general property of the model in that slightly different values for the other parameters also lead to a bifurcation and give similar equilibrium values for C_*. Bifurcations also occur when parameters other than α are varied.

When α lies between 0.06 and 0.105 there are two stable equilibrium levels of crime, the lower level of crime is in the region of 5 to 10 per cent and the higher level ranges from 25 to as much as 35 per cent. It is the initial values of the solution that determine which equilibrium is reached. For example, if α is close to 0.06 then provided the initial value of C is less than about 20 per cent, the solution will tend towards the low crime equilibrium. But when α is nearer to the upper limit of 0.105 an initial value that is only a small distance from the low crime equilibrium would result in the solution tending towards the high level of crime.

This is best viewed in terms of the position of the initial value (S(0), C(0)) in the triangular region T. The equilibrium points and saddle points move about in T as the parameter α is varied (figure 3 shows the C-coordinate of these movements). When α is near 0.06 the high crime equilibrium and saddle point lie close together. As α is increased the saddle point moves away from the high crime equilibrium and closer to the low equilibrium. Broadly speaking, if α is such that the initial value lies between the saddle point and the low equilibrium, the solution will tend to the low equilibrium and similarly for the high equilibrium.

We can determine how much of the flow from S into C is due to the net effect of social and economic conditions and incentives, and how much is due to social interaction. For any values of S and C, the two components of this flow are given by αS and λSC respectively. At the equilibrium point given above with $N_* = 0.80$, $S_* = 0.16$ and $C_* =$

0.04, they are approximately 0.0080 and 0.0045. Given the low proportion in the Criminal category in this particular example, the relative size of the two is not unreasonable. When the parameter α is increased to 0.08 there are two equilibrium points. The one with the higher value of C_* has $N_* = 0.58$, $S_* = 0.11$ and $C_* = 0.31$, a very high proportion of criminals such as might be found in certain inner-city neighbourhoods. In this case, the value of αS_* is 0.0087, very similar to the low equilibrium solution above, but the value of $\lambda S_* C_*$ rises to 0.0239, indicating the crucial importance of the social interaction terms in high crime solutions. The flows from S to C in such circumstances are largely due to the social interaction term, to agents being influenced by the behaviour of those who are already criminals.

6. Conclusion

We have proposed in this paper an approach towards understanding the dynamics of crime which is similar to that used in mathematical biology to model the spread or containment of epidemics. A population, however defined, can be split conceptually at any point in time into three groups, those who are not interested at all in committing crime, those who are susceptible to become criminals, and those who are criminals. A system of differential equations is set up to model the flows between these groups over time.

The strengths of the flows can be interpreted as corresponding to the main factors identified in the empirical literature as the causes of crime, such as demographic movements, general social and economic conditions, and the positive and negative deterrence effects of the criminal justice system. Despite a voluminous literature, no firm quantitative conclusions have emerged, and the model can be used to explore the consequences for crime of variations in the respective strengths of such factors. In principle, the model could be calibrated on a data set of particular crime rates

A key feature of the model is the effect of social interaction between agents, whose existence is widely documented in the general literature of sociology. Social interaction is postulated to operate in two ways. First, the greater the proportion of agents in any given population who are already criminals, the more likely it is that any individual will convert into becoming a criminal at any point in time. Second, the greater the proportion of the population who are wholly disinterested in being criminals, the greater the pressure on those who are criminals to become law-abiding.

The properties of the model are compatible with the key stylised fact about crime, namely that variations in the level of crime across time and space are much greater than the variations in variables such as social and economic conditions which might be thought of as potentially important determinants of crime.

The social interaction terms generate considerable nonlinearities in the level of crime associated with different combinations of the parameters, a point which might underlie the wide differences which are reported in the empirical literature on the effects of various factors on crime.

The social interaction factors also give a clear guideline for policy. The bifurcations in the model, and hence the existence of very high crime equilibria, depend upon these factors. Policies which reinforce respectable community values and which provide strong, non-criminal role models for those agents who are at any point in time most susceptible to commit crime could have quite dramatic effects in reducing crime levels in high crime areas, as well as preventing explosions of crime in relatively low crime areas.

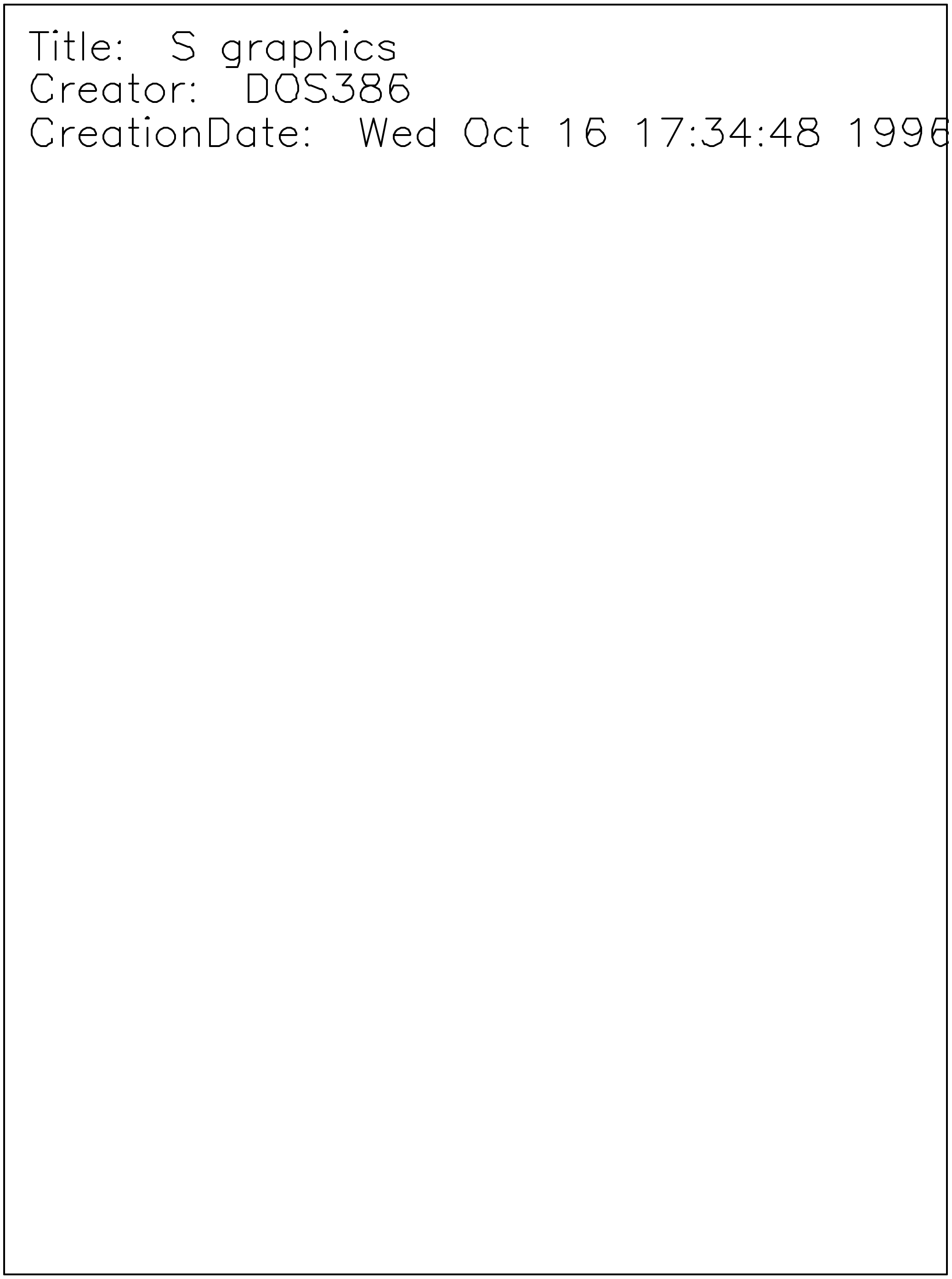

Figure 1. Plot of $\beta(N)$ againts N.

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Figure 2. Plot of $\gamma(N)$ against N for two different values of the parameter α.

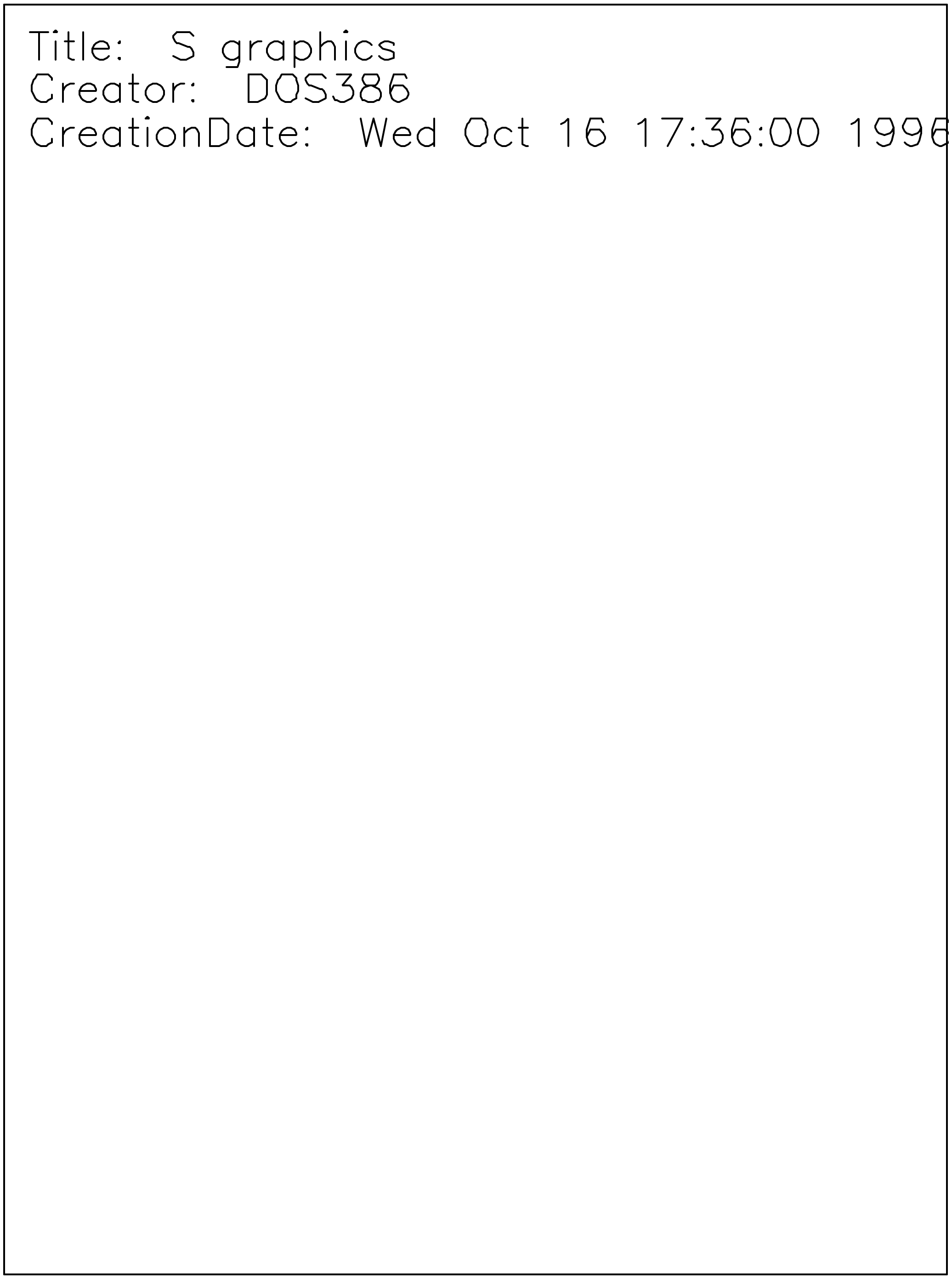

Figure 3. The equilibrium values of C_* for a bifurcation as α is varied.

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