The evolution of family structures in a social context

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1. Introduction

Recent decades have seen major changes in the structure of families in most Western countries. In most countries, both the number of single, never married people and of the divorced as a percentage of the adult population have risen, and the proportion of those who are married has fallen.

A wide range of factors has been variously suggested as being responsible for the changes which have taken place. According to the neoclassical model of Becker et.al. (1977), changes in economic incentives through, for example, the tax and benefit system, and the increased independence of women brought about by rising real incomes and female labour force participation represent potentially powerful explanations.

A large and inconclusive empirical literature exists on the general validity of the Becker thesis. Ermish (1996), for example, does note a correlation across countries between the rate of female labour force participation and the level of divorce, but, as ever, correlation does not prove causation. Haskey (1996) points out the marked trend in the UK for divorce to occur at shorter marriage durations with, for example, only 1 per cent of the 1956 marriage cohort divorced during the first five years of marriage, compared to 13 per cent of the 1986 cohort. This evidence is certainly consistent with the view that the increased economic independence of women enables a broader search procedure for a partner to be carried out, to the extent of being willing to break up rapidly an unsatisfactory relationship.

Oppenheimer (1995) contests this view, suggesting that the post-war American experience of increased female labour force participation with more stable marriages and larger family size offers evidence against the theory, though this is contradicted by England and Farkas (1986) who use American attitudinal data from the late 1950s and 1970s to conclude that divorce becomes more readily 'permitted' when women have independent earnings. Blossfeld et.al. (1995), using data from nine countries on the links between women's educational experience and marriage and family formation, also claim to find that the evidence is not consistent with the Becker hypothesis, with the recent changes in marriage patterns indicating that it is postponement rather than rejection of marriage which is the majority experience

A striking feature of the evidence across OECD countries, however, is the sheer diversity of experience of the different countries in terms of family formation. Whilst the general direction of trends towards more single and divorced people and fewer marrieds is consistent across countries, at any point in time the structures differ considerably (Kuijsten, 1996). In the mid-1990s, for example, the divorce rates in Denmark, Finland and the UK were some 25-50 per cent higher than in France and Germany and more than double the rate in Italy.

Given the very strong degree of overall economic similarity between such countries, reflected for example in income per head, the patterns are consistent with the hypothesis that non-economic, attitudinal factors also play a significant role in patterns of family formation. More generally, much of the empirical literature refers to the potential importance of these factors, whilst at the same time pointing out the very considerable difficulties of gathering such evidence in a practical form.

The aim of this paper is to develop an analytical framework for examining the dynamics of the evolution of family structures at the aggregate level which combines both economic and attitudinal elements. In other words, movements of an individual agent from one type of family structure to another are postulated to depend upon two sets of factors. These are, first, the net effect of the various economic incentives which exist and, second, social attitudes towards marriage and divorce.

An important feature of the model is that the strength of the attitudinal factors is endogenous. The greater the attachment of society to the concept of marriage, the more likely it is for any given level of economic incentives that an individual will, for example, stay married rather than become divorced. But the strength of this attachment is itself measured at any point in time by the proportion of the relevant population which is married.

This feedback mechanism within the model creates non-linearities, which in turn lead to multiple equilibria. The consequence of these is that the same level of overall economic incentives can be associated with different patterns of family structure. In other words, the attitudinal factors give rise to properties of the model which are entirely consistent with a key stylised fact about family structures in the West. Namely, that countries at broadly similar levels of real income per head nevertheless exhibit considerable variation in their family structures.

Section 2 sets out the theoretical model. In section 3, gives an illustrative calibration of the model against the family structure of the UK in the mid-1990s. In section 4, we examine the sensitivity of the results of the model to varying the parameters from their calibrated values. Finally, an Appendix outlines the technical details of the mathematical analysis.

2. The theoretical model of the evolution of family structures

The family structure of any population can be thought of as being made up of a set of stocks and flows. The stocks represent the numbers of individuals within each category or type of family in any given period, and the flows the changes in the stocks, such as the numbers who move from married to divorced, during that particular period.

Schematically, the essence of the model can be set out very simply:

S ------> M <--------> D

in other words, there is a flow from S to M, a flow from M to D and a flow back from D into M. An implication of this is that divorced people re-marrying only marry other people who are divorced, and not singles. This simplifying assumption reduces the analytical complications of the model, which, as we shall see, are already surprisingly considerable.

We abstract from the widowed category, which could easily be added but would not alter the important dynamics of the model. We also abstract from the co-habiting category, so that the singles category contains both those who in practice are genuinely single and those who are co-habiting. Murphy (1996) argues that in the UK increases in cohabitation have partially offset the decline in marriage, and Haskey (1995) shows that premarital cohabitation is now a normal lifecourse stage in Britain. Berrington and Diamond (1995), however, demonstrate that as far as the UK is concerned, cohabitation is a short-lived phenomenon. It does not seem unreasonable to omit what is in general a purely transitory, short-term state between single and married status (although if the model were to be applied to some of the Scandinavian countries, for example, an explicit cohabiting category would become a desirable feature).

A natural way of analysing any system involving stocks and flows is to try to capture its key elements in a set of differential equations. The basic idea of our model is that at any point in time an agent can be in one of a number of potential states, namely single, married or divorced. In the first instance, we describe the aggregate flows between such categories. Even when the system as a whole is in equilibrium, there are still flows of individual agents between the states. The flows depend upon factors such as the severity or otherwise of the divorce laws and economic incentives given, for example, by the level of female real incomes.

But, in addition, we introduce terms to capture the attitudinal process of social emulation, or social interaction. In orthodox economic theory, the tastes and preferences of individuals are fixed, and agents carry out maximising behaviour given their preferences. However, we introduce terms which allow for the behaviour of any agent to be altered directly by observing the behaviour of others.

Specifically, the probability of any individual switching from single to married is related to the proportion of the population who are already married, and the probability of married individuals becoming divorced is also related to the proportion of the population who are divorced. In this way, for example, any given change in the overall economic incentive to marry could have quite different effects in two countries identical in every respect except for the initial pattern of family structure prevailing when the change is made.

Consider in the first instance the flow from M to D. At the level of the individual agent, both the age of marriage and the marital experience of one's parents are known to be good predictors of the probability of divorce (see, for, example, Kiernan (1986, 1997) and Berrington and Diamond (1997)). But these proximate predictors are themselves dependent upon a complex mix of socio-economic and attitudinal factors.

A particular factor to consider is the effect of more liberal divorce laws, though Becker (1991, 1993), for example, argues that such legislation simply reflects economic and social change and does not have a causal impact. Once again the conventional empirical evidence is inconclusive as the discussion on the US evidence in Allen (1992) and Peters (1986, 1992) indicates. Smith (1997) considers evidence for the UK and finds that changes in family law have had in general a powerful but temporary effect on divorce rates, though there is evidence of permanent effects for changes which have particularly affected the transactions costs of marriage.

The flow from M to D is therefore postulated to depend in principle upon the net overall effect of economic factors and changes in the divorce laws. In other words, we are not attempting to identify the precise contribution of any single one of the potentially wide range of variables which might influence, for example, the economic decision to move from married to divorced. Instead, we incorporate the net impact of such factors, whatever it might be and how it might arise, into our model.

These effects are assumed, in the spirit of simplicity and in the absence of decisive evidence to indicate otherwise, to be simply proportionate to M.

The second key factor which we include in the determinants of the flow from M to D is that of social customs or attitudes. This is represented initially by the simple assumption that the flow is proportional to MD ie: the more who are divorced in a given population, the more likely it is that in any period an agent will convert from M to D. More realistically, this relationship is itself likely to be non-linear. If only a very small proportion of the population is in D, any change in this is unlikely to have much of an effect on the flow from M to D. Similarly, if a very substantial proportion is in D, small changes will not have much effect. In other words, this relationship is likely to be S-shaped eg: a logistic-type curve. But this complication is avoided for the moment. So we have:

$$dM/dt = -\alpha M - \lambda MD \qquad (1)$$

The flow out of D back into M is assumed, in the first instance, to be simply proportional to D

$$dD/dt = -\theta D \qquad (2)$$

The flow from S to M is again postulated to depend upon two factors:

- the net effect of economic factors such as the general level of tax and benefit incentives and the economic independence of women given by the level of real incomes and the rate of female labour participation

- social customs

The first of these is assumed, for simplicity, to be proportional to the numbers in the S category.

The second is again represented by the simple assumption that the flow is proportional to SM ie: the more who are married in a given population, the more likely it is that in any period an agent will convert from S to M, though more realistically, this relationship is itself again likely to be non-linear.

So we have:

$$dS/dt = -\mu S - \delta SM \qquad (3)$$

As the model stands, S will eventually be emptied. We therefore need to take account of the fact that the S category is replenished by births. Assume births and deaths are proportionate to the size of the relevant category. Assume further that the overall population is static. This latter is not a bad approximation to reality in Europe, and has the advantage of simplifying the analysis considerably.

Obviously, no-one is born married or divorced. So, in this basic form, the model can be written with the net growth in the population of S being equal to the fall in the population in M and D. In other words, the M and D equations could be augmented by, respectively, $-\phi_2 M$ and $-\phi_3 D$, and the S equation by $(\phi_2 M + \phi_3 D)$. In practice, death rates amongst the married and divorced are fairly similar, so, in the spirit of keeping the model as simple as possible in order to understand the key dynamics, assume that $\phi_2 = \phi_3 = \phi$.

We can now pull the various bits together:

$$dS/dt = -\mu S - \delta SM + \phi(M + D) \qquad (4)$$

$$dM/dt = \mu S + \delta SM - \alpha M - \lambda MD - \phi M + \theta D \qquad (5)$$

$$dD/dt = \alpha M + \lambda MD - \phi D - \theta D \qquad (6)$$

As with any system of differential equations, there are several key tasks in analysing the system described by equations (4) - (6):

i) to verify that in any solution of the equations $S + M + D$ is constant and equal to the total population. This condition is required for the model to have meaningful solutions.

ii) to establish the existence or otherwise of equilibrium values (ie: values of S, M and D such that $dS/dt = dM/dt = dD/dt = 0$) for which S, M and D are positive.

iii) to establish the stability properties of the equilibrium values.

By substituting the expression for D derivable from equation (6) into equations (4) and (5), a cubic equation can be obtained. In other words, an explicit analytical solution of the system exists. The actual solutions involve long combinations of the parameters of the model which it would be very tedious to list, although details are available from the authors[1]. However, in summary, we can say that, first, the model has three equilibrium solutions. Second, that only one of these solutions satisfies the conditions in (ii) above. Third, that this solution is an attracting one. In other words, the equilibrium solution in which we are interested is stable.

However, as mentioned above, it is more realistic to consider a system in which the parameters δ and λ are themselves endogenous. Equation (4), for example, becomes

$$dS/dt = -\mu S - \delta(M)SM + \phi(M + D) \qquad (7)$$

Similarly, equation (5) becomes

$$dM/dt = \mu S + \delta(M)SM - \alpha M - \lambda(M)MD - \phi M + \theta D \qquad (8)$$

and equation (6)

$$dD/dt = \alpha M + \lambda(M)MD - \phi D - \theta D \qquad (9)$$

where $\delta(M)$ and $\lambda(M)$ are non-linear, S-shaped functions[2].

3. Calibration of the theoretical model

The next step is to try to parameterise the model so that the solution in which we are interested gives results which are similar to those of a real life situation

The past forty years have seen dramatic changes in family structure in the UK. Comparing the early 1960s with the mid-1990s, the number of divorced people has risen from around 400,000 to some 3 million. The population of singles of marriageable age (ie: never married over 18) has grown from some 10 to 13 million, whilst the number of married people has remained constant at around 27 million. Taking these as a percentage of the total of the three categories, the percentage of singles has risen from 27 to 31, of divorced from 1 to 7, whilst the married percentage has fallen from 72 to 62. These figures, whether in level or percentage form, can be thought of as forming the stocks of S, M and D in our model.

In terms of annual flows (ie: the dS/dt, dM/dt, dD/dt terms), in the early 1960s there were just under 400,000 marriages a year. In other words, around three-quarters of a million people got married each year. By the mid-1990s, the number of marriages had fallen to some 330,000, despite the growth in the population. The number of divorces was 30-40,000 a year in the early 1960s, and is now around 170,000 a year.

A substantial proportion of these changes took place in the decade from the late 1960s onwards and, although the figures are not completely constant, the 1990s have seen less dramatic changes. As an *approximation*, we can regard the 1990s as being an equilibrium outcome (ie: with dS/dt, dM/dt, dD/dt all approximately zero). We are therefore looking to choose parameters which lead to *approximately* the stocks and flows which we

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1 The program *Mathematica* was very helpful in analysing this system

2 The precise form adopted was........, where h........, v.........

observe in actual data for the UK in the mid-1990s. In principle, of course, the model could be calibrated against any set of data which could be reasonably represented as stable.

The model cannot be calibrated exactly against 1990s UK data, because the actual situation is only an approximation to an equilibrium one. An important reason for this is that, although population growth is low, it is not actually zero. Overall, there are between 700 and 750,000 births each year and 300 -350,000 deaths. This has repercussions on our ability to produce numerical results which are almost identical to the actual data not only for S, M, D and dS/dt, dM/dt, dD/dt, but for the various sub-component flows within the latter. However, we are able to get close enough to then use the model to examine the impact of parameter - and by implication, policy - change.

The parameter most obviously affected by the low but non-zero population growth is ϕ, which in the model represents both the birth and the death rate. Taking the average of the two actual rates of 525,000 a year, it is easy to see that this should equal, in the model, $\phi(M + D)$. Given that $M + D$ is just under 30 million, the implied value of ϕ is 0.018. (The precise value of the stocks we choose is S at 13.0 million, M at 26.6 million and D at 3.0 million).

The parameter θ represents the re-marriage rate out of the stock of the divorced. We assign a value of 0.06, which implies a total number of re-married individuals of 0.06x3mn = 180,000 a year, a figure which is somewhat higher than in practice. The flow from S into M is given by $(\mu + \delta M)S$. If we assume that 530,000 single individuals get married each year, and dividing this by 13 million (ie: the value of S), calibrating the model in units of a million, we obtain

$$\mu + \delta M = 0.041$$

The total number of marriages in the parameterised version of the model is therefore 710,000, slightly higher than the actual data, but we have the property that the rate of marriage from the divorced stock is greater than that out of the stock of singles, which is consistent with the evidence.

The flow from the M to the D category each year is some 340,000 individuals. In the model, the flow out of M is given by $(\alpha + \lambda D)M$.

Obviously, there is a wide range of values which could be used for μ, δ, α and λ to satisfy these two relationships. But we can build up reasonable values step by step. In the absence of evidence to the contrary, for example, it seems reasonable to assume that the two parameters which determine the extent of social interaction, δ and λ, should be equal.

Further, there is no obvious reason why the strength of social interaction effects should have altered over the past thirty years or so. There were very few divorced people in the 1960s, but the subsequent change is much more plausibly attributed to changes in the value of α, representing the net effect of economic impacts and divorce laws, than it is to the idea that individuals might suddenly have become far more susceptible to the influence of others.

This latter argument sets quite strong upper limits on λ, which we assign at 0.0005 and, on the previous argument, give δ the same value. In turn, this choice implies a value of μ of 0.028, given a value of M of 26.6. In other words, around two-thirds (0.028/0.041) of the flow from S to M is attributed to the net effect of economic factors, and one-third to social custom/interaction. Since the UK data for the mid-1990s do not represent an exact equilibrium situation, we need to introduce a certain amount of leeway, as it were, into the choice of α, which we set at 0.0080. (Given our choice of λ and the actual values of M and D, the parameter would take the value 0.0100).

In summary, the parameters are set at the following values:

$\alpha = 0.0080$; $\delta = 0.0005$; $\lambda = 0.0005$; $\mu = 0.028$; $\phi = 0.018$; $\theta = 0.0060$

which leads to the solution S = 12.95, M = 26.4, D = 3.25, which is very close to the actual values of, respectively, 13.0, 26.6 and 3.0.

In the model, the terms δSM and λMD introduce non-linearities which complicate very considerably the analysis of the model. However, in the immediate neighbourhood of the values of δ and λ which are chosen to calibrate the model numerically, relationships are only moderately non-linear. We therefore examined the sensitivity of the analytical properties of the model when a more explicit S-shaped functional relationship is used to represent the social custom factors.

Using the flow from S to M as the example, in the model as it stands, the social custom term is given by δSM. To introduce a stronger degree of non-linearity, we make δ a function of M, so that the term becomes $\delta(M)SM$. The choice of the tanh functional form, suitably scaled, gives the kind of S-shaped relationship for which we are looking. The point of inflexion of the curve and the slope in its neighbourhood can be set as parameters.

Introducing this kind of relationship makes the analysis of the system even more complicated, and an outline of the approach adopted is set out in the Appendix.

Essentially, we are interested in whether the use of the more non-linear social custom terms changes the analytical properties of the system. In the model given by equations (4) - (6), there is a single attracting equilibrium solution, which also satisfies the conditions that S, M and D are all positive.

In summary, it is possible to choose $\delta(M)$ such that a second attracting equilibrium solution is created, and which also satisfies the requirements of common sense set out in the previous paragraph. An implication of this is that, depending upon how close any given values of, say, α and μ are to the point of bifurcation, small changes in these parameters can lead to relatively large changes in the values of S, M and D, as the system moves from the basin of attraction of one of the stable solutions to the other.

4. Sensitivity analysis of the calibrated model and its implications

Table 1 summarises the effect on the equilibrium solution of varying, in turn, α and μ from their values in what we can call the 'base solution' set out at the end of paragraph 8. We examine the effect of halving the values of each of the parameters, of increasing them by 50 per cent, and also of doubling them.

It is important to note that we are not saying how, in practice, the impact of the net effects summarised by each of these parameters can be halved or doubled, or whatever. We are examining the implications *if* changes leading to effects of this magnitude on the parameters could be carried out.

Table 1. Impact of varying a and m, separately, from values in base solution

	S	**M**	**D**
base solution	12.95	26.4	3.25
2α	13.25	23.6	5.7
1.5α	13.1	24.9	4.55
0.5α	12.75	28.05	1.75
2μ	8.6	30.2	3.85
1.5μ	10.3	28.7	3.6

0.5μ	17.8	22.15	2.65

The parameter α represents the net impact of economic factors and the divorce laws on the flow from M to D. The main impact on the equilibrium values of the stocks when α is altered is therefore on M and D, though S also changes through the δSM term because M itself changes. Similarly, changes in μ mainly affect S and M, but also D through the λMD term.

One way of gaining a perspective on the simulations is to consider what the sizes of S, M and D would be if the percentage share of each in the total which existed in the early 1960s obtained in the mid-1990s. There would now be approximately 11.5 million people in the S category, 30.5 million in M and 0.5 million in D.

This example is *not* used, it must be emphasised, because we wish in some way to 'turn the clock back' and restore the overall social situation which existed in the early 1960s. But, rightly or wrongly, much of the policy discussion contrasts family outcomes now with those which prevailed in the 1950s and 1960s. These figures should be seen as a benchmark with which to judge how easy or difficult it might be for policy changes to bring about a similar outcome now, without necessarily taking a view on the desirability of such an outcome.

The parameter μ summarises the net effect of economic factors on the flow from S to M. Table 1 suggests that the changes required to encourage sufficient numbers to move out of S to reduce the total to 11.5 million are not absolutely enormous. An increase in μ of around 30 per cent would be sufficient.

However, the effect of varying α is such that it appears to be impossible to reduce the numbers in the divorced category (by this route alone) without making truly dramatic policy changes, such as very marked tightening of the divorce laws. Halving the effect of α does have a substantial impact on the numbers in D, reducing them by some 1.5 million, or by almost 50 per cent. But there would still be 1.75 million people in the D category, in contrast to the 0.5 million who would be in it if the proportions of the 1960s obtained now.

It is easy to think of the kinds of policies which would change α and μ, both up and down. It is more challenging to imagine how the social custom parameters, δ and λ might be altered, although the present government appears to believe that public exhortation is one way in which this might happen. Table 2 sets out the impact of varying these two parameters on the equilibrium solutions.

Table 2. Impact of varying d and l, separately, from values in base solution

		S	M	D
base solution ($\delta = \lambda = 0.0005$)		12.95	26.4	3.25
d	0.0001	15.85	23.85	2.9
	0.0010	10.25	28.75	3.6
	0.0020	7.0	31.5	4.05
l	0.0001	12.90	26.85	2.85
	0.0010	13.05	25.65	3.9
	0.0020	13.3	23.35	5.95

The results of Table 2 are in a key respect similar to those of Table 1. Namely, that encouraging the flow from S to M appears to be easier to bring about than discouraging flows from M to D. In fact, there is very little to be gained by even a dramatic reduction in the social custom parameter, λ, in terms of reducing the numbers in the D category.

A clear implication of the sensitivity analysis is that the main agent of change to family structures in the UK has been changes in the parameter μ and, to some extent, through the changes in divorce law, α. In other words, that the changes are to a considerable extent due to the impact of rising real incomes and labour force participation on the economic independence of women

5. Conclusion

The conventional empirical literature on the causes of changes in family structures is extensive but inconclusive. For example, some studies offer evidence which is consistent with the hypothesis that an important reason for the rise in divorce in the West is the increased economic independence of women, whilst others suggest that it should be rejected.

Much of this literature suggests that attitudinal factors also play a significant role in patterns of family formation, whilst at the same time pointing out the very considerable difficulties of gathering such evidence in a practical form.

In this paper, we develop an analytical framework for examining the dynamics of the evolution of family structures at the aggregate level which combines both economic and attitudinal elements. The family structure of any population can be thought of as being made up of a set of stocks and flows. The stocks represent the numbers of individuals within each category or type of family in any given period, and the flows the changes in the stocks, such as the numbers who move from married to divorced, during that particular period.

A natural way of describing any system involving stocks and flows is to try to capture its key elements in a set of differential equations. The basic idea of our model is that at any point in time an agent can be in one of three potential states, namely single, married or divorced (It is possible to generalise the model to introduce other categories such as co-habitation). The flows between the various states are determined by the overall net impact of economic factors, such as the tax and benefit regime, female real wages and labour force participation, and institutional changes such as modifications of the divorce laws.

In addition, we introduce terms to capture the attitudinal process of social emulation, or social interaction, which allow for the behaviour of any agent to be altered directly by observing the behaviour of others. Specifically, the probability of any individual switching from single to married is related to the proportion of the population who are already married, and the probability of married individuals becoming divorced is also related to the proportion of the population who are divorced.

We do not make any explict assumptions about the impacts of the various potential determinants of flows between the different family types, but simply include their overall, net effects as parameters into the model.

The social interaction terms introduce non-linearities into the model, but nevertheless it has a single, attracting equilibrium solution. The data for the UK in the 1990s can be regarded as an approximation to an equilibrium, in that the numbers in each of the three categories in our model is approximately constant (ie: their rates of change are approximately zero). We therefore calibrate our model against this data.

We carry out sensitivity analysis of the equilibrium solution of the model by varying the values of the parameters from their calibrated values. Specifically, we investigate the magnitude of changes in the parameters which would be required to lead to a solution similar to that which prevailed in the UK in the early 1960s. A clear implication of the sensitivity analysis is that the changes in family strucuture in the UK over the past three decades or so are to a considerable extent due to the impact of rising real incomes and labour force participation on the economic independence of women.

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Appendix

To simplify the analysis, we interpret S, M and D as proportions of the population, and complete the system with the identity

$$S + M + D = 1 \qquad (7)$$

This is used to eliminate D from equations (4) and (5).

The two-equation model we analyse is

$$dS/dt = -\mu S - \delta SM + \phi(1 - S) \qquad (8)$$

$$dM/dt = \mu S + \delta SM - \alpha M - \lambda M(1 - S - M) - \phi M + \theta(1 - S - M) \qquad (9)$$

A further simplification was to normalise the parameters with respect to ϕ, the birth and death rate. By setting ϕ equal to 1, a parameter is eliminated.

In order to be able to use the model, we require that the solutions S(t) and M(t) to equations (8) and (9) both lie between 0 and 1 and in addition that S(t) + M(t) is less than 1 so that D(t) = 1 – S(t) – M(t) also lies between 0 and 1. This can be summarised neatly in a geometrical way by saying that the point (S(t), M(t)) must lie in the triangular region T of the plane defined by

$$T = \{(S, M) : 0 \leq S \leq 1, 0 \leq M \leq 1 \text{ and } 0 \leq S + M \leq 1\}$$

for all values of t.

Any system of differential equations, such as (8) and (9), is equivalent to a vector field. In this case, the vector field associates the point with co-ordinates (S, M) in the plane with the vector (dS/dt, dM/dt). A solution of the equations (S(t), M(t)) traces out an unbroken curve in the plane as t increases. It has the property that at each point (S, M) on the curve, the tangent to the curve is the value of the vector field at that point, i.e. (dS/dt, dM/dt).

Viewing the equations as a vector field it becomes clear that in order to show that a solution with valid initial values remains in the region T, we simply need to verify that at each point on the boundary of T the vector field given by (dS/dt, dM/dt) points towards the interior of T. This ensures that a solution that starts in the region T stays inside T since it can not cross its boundary.

The stability properties of an equilibrium point can be determined by making a linear approximation to the actual equations and then examining the stability of the linear approximation.

Denote an equilibrium solution of (8) and (9) by (S_*, M_*). We need first to make a linear approximation to the vector field that holds good in the neighbourhood of (S_*, M_*). The vector field can be regarded as a map from two dimensional space to two dimensional space. The derivative of this map evaluated at the point (S_*, M_*) gives the linear approximation we are looking for. The derivative can be written as a matrix A. The derivative of the vector field involves the partial derivatives of each of the right hand sides of equations (8) and (9) with respect to the variables S and M.

The real parts of the eigenvalues of the matrix A show directly the stability properties of the solution.

Finding the equilibrium points for the equations in which either/or both δ and λ depend upon the values of, respectively, M and D is considerably more difficult than the case discussed above.

Consider by way of example letting λ depend upon the value of D. The point (S_*, M_*) will be an equilibrium point for equations (8) and (9) if it happens to coincide with the equilibrium point for the case where λ is constant and takes the value $\lambda = \lambda(1 - S_* - M_*)$. We can use a graphical method to find the point where they coincide.

Suppose for the moment that D is fixed. The value of D determines the value taken by the function λ. For any constant λ we can find an equilibrium point (S_*, M_*). Let $(S_*(D), M_*(D))$ be the equilibrium point for the value of λ given by D. The value of D also determines a line in T, i.e. $D = 1 - S - M$ can be rearranged to give M in terms of S,

$$M = (1 - D) - S$$

which is the equation of a line with a slope of -1 intercepting the M-axis at $1 - D$. We have found an equilibrium point if $(S_*(D), M_*(D))$ lies on this line.

We define a function γ to measure the vertical distance between the point and the line,

$$\gamma(D) = M_*(D) - ((1 - D) - S_*(D)).$$

By plotting $\gamma(D)$ against D, the values D_* for which $\gamma(D_*) = 0$ can be found. The equilibrium points are then the points $(S_*(D), M_*(D))$ corresponding to $D = D_*$.